

Inelastic diffraction and meson radii

Meng Ta-chung, R. Rittel, K. Tabelow, and Zhang Yang*

Institut für Theoretische Physik, FU-Berlin, 14195 Berlin, Germany

e-mail: meng@physik.fu-berlin.de; rittel@physik.fu-berlin.de; tabelow@physik.fu-berlin.de;

zhangy@itp.ac.cn

Abstract

It is shown that meson radii can be extracted from diffractive scattering experiments of the type: $m + p \rightarrow m + X$, where p stands for the target proton, and m for the incident charged meson π^+ , π^- , K^+ , or K^- . The basis of the method used to perform this extraction is the QCD-based SOC-picture for inelastic diffraction in high-energy hadron-hadron and lepton-hadron collisions proposed in a recent Letter. The obtained results for pion and kaon charge radii are compared with those determined in meson form factor measurements.

Ever since it is known that not only nucleons, but also pions and kaons, are spatially extended objects, determination of meson radii has been of considerable interest — both experimentally and theoretically (see e.g. Refs.[1-12] and papers cited therein). Independent of the fact whether the collected information on meson radii is obtained from electroproduction¹⁻³, from e^+e^- -annihilation^{4,5}, or from direct meson-electron scattering⁶⁻¹² experiments, such information have always been extracted from meson form factors — based on Feynman graphs (or subgraphs) in which the meson is scattered elastically in an electromagnetic field. Can we determine the spatial extension of a meson without any knowledge of its electromagnetic form factor?

**Present address:* Chinese Academy of Sciences, Institute of Theoretical Physics, POB 2735, Beijing 100080, China

In this Brief Report, we show that meson radii can be extracted from inclusive differential cross-section data for diffractive scattering processes of the following type: $m + p \rightarrow m + X$ where p stands for the proton target and m is the incident meson: π^+, π^-, K^+ , or K^- . The method used for this extraction is a QCD-based SOC-picture for inelastic diffraction in high-energy hadron-hadron and lepton-hadron collisions proposed in a recent Letter¹³, in which the following are pointed out:

First, systems of interacting soft gluons which play the dominating role in diffractive scattering processes are extremely complex — so complex that concepts and methods developed for the description of Complex Systems, in particular those in connection with the existence of Self-Organized Criticality^{14–16}, are needed in understanding the observed phenomena. The applicability of such concepts and methods are justified, and the existence of self-organized criticality (SOC) can be expected, because of the following: (A) The characteristic properties of the gluons (especially the direct gluon-gluon coupling prescribed by the QCD-Lagrangian, the confinement, and the non-conservation of gluon-numbers) strongly suggest that systems of interacting soft gluons are open, dynamical, non-equilibrium, complex systems with many degrees of freedom. (B) It is this kind of systems which has been intensively studied in Sciences of Complex Systems, and it is in this kind of systems where the fingerprints of SOC have been found. (C) The existence of SOC-fingerprints can be, and have been, checked^{13,17} in systems of interacting soft gluons by examining high-energy scattering processes in which such gluons dominate. It is observed in particular that the “colorless objects” which play the dominating role in inelastic diffractive scattering processes can be considered as color-singlet gluon-clusters in form of BTW-avalanches^{14–16} due to SOC in systems of interacting soft gluons. To be more precise: Such colorless gluon-clusters are in general partly inside and partly outside the proton. That is, they form a kind of “cluster cloud”^{13,17}. Since the average binding energy between color-singlet aggregates is of Van der Waals type, which implies that this kind of binding energy is negligible small compared with the corresponding one between colored objects, it is expected that, even at relatively small values of momentum-transfer ($|t| < 1 \text{ GeV}^2$, say), the struck colorless clusters can be

absorbed by the beam-particle, and thus “be carried away” by the latter.

Second, optical-geometrical concepts and methods can be used to examine the space-time properties of the above-mentioned colorless objects. In fact, having the well-known phenomena associated with Fraunhofer’s diffraction and the properties of de Broglie’s matter waves in mind, the projectiles P (γ^* , γ , \bar{p} , or p) in a scattering process

$$P + T \rightarrow X + T, \quad (1)$$

where T stands for the target proton (p), P stands for the projectile which can be an antiproton (\bar{p}), a proton (p), a virtual photon (γ^*), or a real photon (γ), that is

$$\begin{cases} P = \gamma^*, \gamma, p, \text{ or } \bar{p}, \\ T = p. \end{cases} \quad (2)$$

can be viewed as high-frequency waves passing through a medium, where the medium is the above-mentioned “cloud” of color-singlet gluon-clusters.

In fact, as we have already pointed out in Ref.[13], the proposed picture for *inelastic* diffraction for high-energy collisions has two ingredients which are fundamentally different from those in the conventional picture for *elastic* diffraction: (i) “The scatterer” in the proposed picture for *inelastic* diffraction is *not* a static object. It is a Complex System which depends on space and time! (ii) In the proposed picture for *inelastic* diffraction, the wave-vector of the outgoing de Broglie wave (X) differs from the incoming one (P), not only in direction, but also in magnitude. That is, the frequency and the longitudinal component of the wave-vector of the outgoing wave (X) can differ very much from their counterparts in the incoming wave (P). This is why, and how, relatively large energy-momentum transfer in passing the medium can result in large invariant mass M_x for X which is described as outgoing waves in such *inelastic* diffraction processes.

Based on the proposed picture¹³ simple analytical formulae for the differential cross-sections $d\sigma/dt$ and $d^2\sigma/dtd(M_x^2/s)$ can be, and have been, derived^{13,17} for inelastic diffractive scattering processes. They are

$$\frac{d\sigma}{dt} = C \exp(-2a\sqrt{|t|}) \quad (3)$$

$$\frac{1}{\pi} \frac{d^2\sigma}{dx_P dt} = N x_P^{-2/3} \exp(-2a\sqrt{|t|}), \quad (4)$$

where C and N are unknown (normalization) constants, which are obviously related to each other, $x_P = M_x^2/s$ (\sqrt{s} is the total cms energy, and M_x is the invariant mass of the unidentified hadronic system X), t is the invariant momentum transfer (which is, for $x_P \ll 1$, approximately $-p_\perp^2$, where p_\perp is the transverse momentum of the scattered particle). The constant a is, because of SOC, QCD, and confinement, directly related (see Ref.[13] and/or Ref.[17] for details) to the proton's charge radius $r_p \equiv \langle r_p^2 \rangle^{1/2}$:

$$a = \sqrt{\frac{3}{5}} r_p. \quad (5)$$

While the derivation of Eqs.(3), (4), and (5) presented in [13] and [17] will not be repeated here, we do wish to emphasize the following: The reason why the charge radius is used in these equations as a measure of proton's confinement region is also closely related with the fact that gluons carry color. To be more precise, the fact that gluons carry color, not only implies that the spatial location where the (colored and colorless) gluon-clusters in form of BTW-avalanches are initiated has to be inside the confinement region of the proton, but also implies the following: In accordance with QCD, the chance for colored objects to create seaquark-antiseaquark ($q_s \bar{q}_s$) pairs is larger than that for a corresponding colorless objects to do so. This means, the created $q_s \bar{q}_s$ -pairs are mainly due to the colored objects — all of which (including the valence quarks and the gluons) have to remain in the confinement region of the target proton. Having in mind, that the electric charge of q_s and that of \bar{q}_s can be detected by an incident (real or virtual) photon, we see that the region in which electric charge can be detected, and the region in which BTW-avalanches in gluon systems can be initiated, are (at least approximately) *the same*. This is the reason why charge radius of the proton plays the important role in Refs.[13] and [17], and this is also why useful information on meson radii can be expected by looking at the corresponding single diffractive scattering processes.

In order to extract meson radii from data for inelastic diffraction in high-energy collision processes, we now examined single diffractive scattering processes of the type

$$P + T \rightarrow P + X, \quad (6)$$

where P is a charged meson and T a proton, that is

$$\begin{cases} P = m = \pi^-, \pi^+, K^-, \text{ or } K^+, \\ T = p. \end{cases} \quad (7)$$

Having the proposed¹³ QCD-based SOC-picture for inelastic diffraction in high-energy scattering processes in mind, the apparent symmetry between the reactions mentioned in Eqs.(1) and (2) and those mentioned in Eqs.(6) and (7) show that they are closely related to one other, as we can explicitly see in Fig.1.

In fact, it follows immediately from Fig.1, that the differential cross-sections $d\sigma/dt$ and $d^2\sigma/dtdx_P$ in the corresponding kinematical regions should have the same kind of behaviors as those shown in Eqs.(3), (4) and (5) with the following modification: The proton radius in Eq.(5) should be replaced by the corresponding meson radius namely by r_π for pions, and by r_K for kaons. It is because, in this case, the struck color-singlet gluon-cluster in form of a BTW-avalanche is initiated *in the meson*, while the proton in the reactions mentioned in (6) and (7) appears in form of a de Broglie wave which undergoes Fraunhofer diffraction. This means, based on the picture proposed in Ref.[13], it should be possible to determine the radius of the charged meson from the inclusive cross section data¹⁸ obtained in the corresponding *inelastic diffractive scattering experiments*.

In order to put this idea into practice, we need to compare these formula with the data¹⁸ and thus extract the “slope parameter” if the agreement is reasonable. Here we have to take the following facts into account: First, according to Eqs.(3) and (4), it is only the dependence of the differential cross section on p_\perp (recall that p_\perp is approximately $\sqrt{|t|}$ for $x_P \ll 1$) which is directly associated with the meson radius. Second, there seems to be no integrated data for $d\sigma/dt$ (or $d\sigma/dp_\perp$) for the considered reactions. Based on these facts we performed a fit to the data¹⁸ for the double differential cross section for $m + p \rightarrow m + \text{anything}$

with $m = \pi^+, \pi^-, K^+, K^-$, where the p_\perp -dependence of the differential cross sections is given by (4) for every given value of $x_P = M_x^2/s$. In these fits, the charges of the pions and those for the kaons are ignored in order to have better statistics. The results of these fits together with the corresponding values for the meson radius r_m are shown in the Figs. 2 and 3, respectively. The value for the radius is obtained by averaging over the values for r_m at different x_P weighted by the number of points (minus one) in each box. The results are

$$r_{\pi^\pm} = 0.62 \pm 0.04 \text{ fm}, \quad (8)$$

$$r_{K^\pm} = 0.53 \pm 0.03 \text{ fm}. \quad (9)$$

A comparison with the results obtained by using other methods is shown in Figs. 4(a) and 4(b).

In addition, we recall that, in order to minimize systematic errors, the difference of the squared radii of pions and kaons $\langle r_\pi^2 \rangle - \langle r_K^2 \rangle$ has been measured and given in Refs.[8] and [12]. The experimental values are $0.16 \pm 0.06 \text{ fm}^2$ in⁸, and $0.10 \pm 0.045 \text{ fm}^2$ in¹², respectively. Our result for this difference extracted from the data given in Ref.[18] is:

$$\langle r_\pi^2 \rangle - \langle r_K^2 \rangle = 0.10 \pm 0.07 \text{ fm}^2 \quad (10)$$

A graphical comparison is shown in Fig. 4(c).

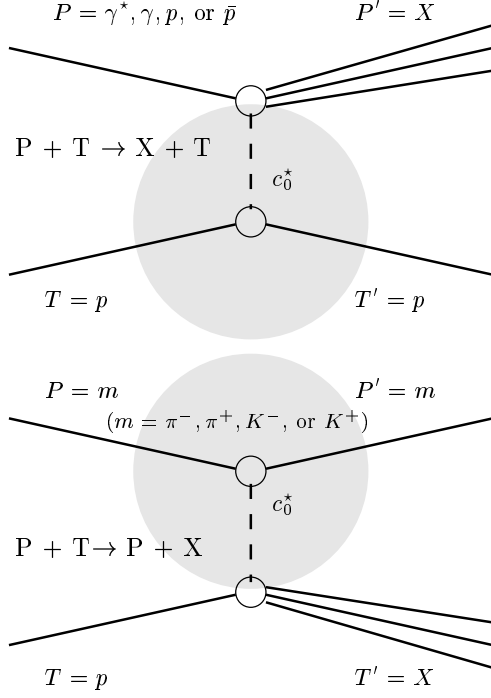
In conclusion, we see that, by applying the physical picture proposed in Ref.[13] to single diffractive scattering processes, in which charged mesons are used as projectile, and produced hadrons are found in the fragmentation region of the proton target, meson radii can be extracted from *inelastic diffractive scattering data*¹⁸. The obtained results for pion and kaon charge radii are in good agreement with those obtained from meson form factor measurements (see Refs.[1-12] and the papers cited therein.). Since this application can be considered as a crucial test of the usefulness of concepts and methods developed for Complex Systems [in particular those in connection with the existence of Self-Organized Criticality], in understanding the mechanisms of reactions where interacting gluons play a dominating role, the results presented in this Brief Report can be considered as further confirmation of the usefulness of this approach.

The authors thank Cai Xu for helpful discussions, and FNK der FU for financial support.
Y. Zhang also thanks Alexander von Humboldt Stiftung for the fellowship granted to him.

REFERENCES

1. S. F. Bereznev, et al., Sov. J. Nucl. Phys. **16**, 99 (1973).
2. C. J. Bebek, et al., Phys.Rev. **D13**, 25 (1976).
3. C. J. Bebek, et al., Phys.Rev. **D17**, 1693 (1978).
4. Quenzer, et al., Phys. Lett. **B76**, 512 (1978).
5. Barkow, et al., Nucl. Phys. **B256**, 365 (1985).
6. G. T. Adylof, et al., Phys. Lett. **51B**, 402 (1974); Nucl. Phys. **B128**, 461 (1977).
7. E. B. Dally, et al., Phys. Rev. Lett. **39**, 1176 (1977); Phys. Rev. **D24**, 1718 (1981).
8. E. B. Dally, et al., Phys. Rev. Lett. **48**, 375 (1982).
9. S. R. Amendolia, et al., Phys. Lett. **146B**, 116 (1984).
10. S. R. Amendolia, et al., Nucl. Phys. **B277**, 168 (1986).
11. E. B. Dally, et al., Phys. Rev. Lett. **45**, 232 (1980).
12. S. R. Amendolia, et al., Phys. Lett. **178B**, 435 (1986).
13. T. Meng, R. Rittel, and Y. Zhang, Phys. Rev. Lett. **82**, 2044 (1999).
14. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); Phys. Rev. **A38**, 364 (1988).
15. P. Bak, *How Nature Works*, (Springer, New York 1996); and papers cited therein.
16. H. J. Jensen, *Self-Organized Criticality*, (Cambridge University Press, Cambridge 1998); and papers cited therein.
17. C. Boros, T. Meng, R. Rittel, K. Tabelow, and Y. Zhang, hep-ph/9905318, submitted to Phys. Rev. **D**.
18. A. E. Brenner, et al., Phys. Rev. **D26**, 1497 (1982).

FIGURES



P is viewed as the incident de Broglie wave in this inelastic diffraction, where X which stands for the system of hadrons in the fragmentation region of P is viewed as the outgoing de Broglie wave.

Except energy-momentum, the quantum numbers of T and T' are the same. The invariant momentum transfer between T and T' is $t \approx p_{\perp}^2$. Since the color-singlet gluon-cluster c_0^* absorbed by P originates from T and appears as part of the cluster-cloud of T , it is the confinement region of T that plays the significant role.

Except energy-momentum, the quantum numbers of P and P' are the same. The invariant momentum transfer between P and P' is $t \approx p_{\perp}^2$. Since the color-singlet gluon-cluster c_0^* absorbed by T originates from P and appears as part of the cluster-cloud of P , it is the confinement region of P that plays the significant role.

T is viewed as the incident de Broglie wave in this inelastic diffraction, where X which stands for the system of hadrons in the fragmentation region of T is viewed as the outgoing de Broglie wave.

Fig. 1. Inelastic diffraction in high-energy collision processes mentioned in Eqs.(1) and (2), and those in Eqs.(6) and (7), viewed in the c.m.s. frame of the projectile (P) and the target (T). The system of hadrons in the final state is a consequence of the (compared to elastic diffraction) relatively large transfer of energy-momentum. The system is viewed as the product of c_0^* and the incident wave.

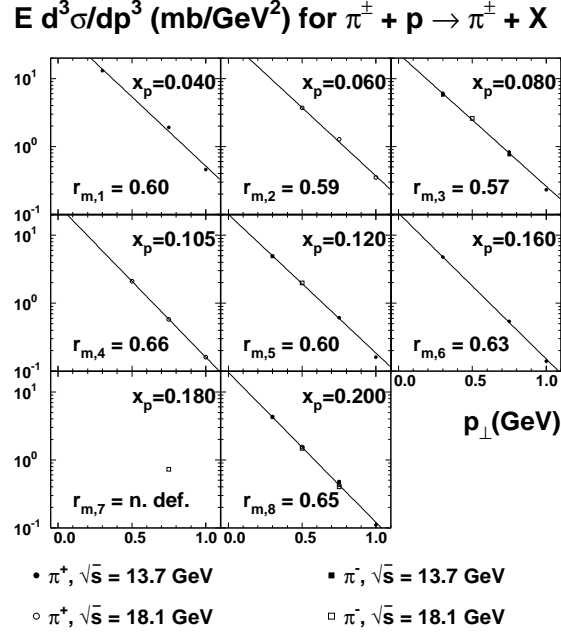


Fig. 2. Double differential cross section for diffractive inelastic $\pi^\pm + p \rightarrow \pi^\pm + X$ scattering as a function of the transverse momentum of the scattered meson at given $x_P = M_x^2/s$. The data are taken from Ref.[18]. The lines are the result of a fit to the data. The pion radius obtained by using the method described in the text is $r_{\pi^\pm} = 0.62 \pm 0.04 \text{ fm}$.

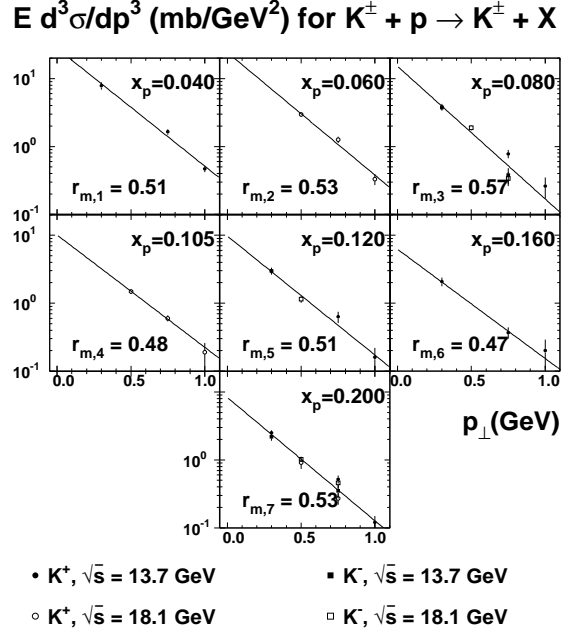


Fig. 3. Double differential cross section for diffractive inelastic $K^\pm + p \rightarrow K^\pm + X$ scattering as a function of the transverse momentum of the scattered meson at given $x_P = M_x^2/s$. The data are taken from Ref.[18]. The lines are the result of a fit to the data. The kaon radius obtained by using the method described in the text is $r_{K^\pm} = 0.53 \pm 0.03$ fm.

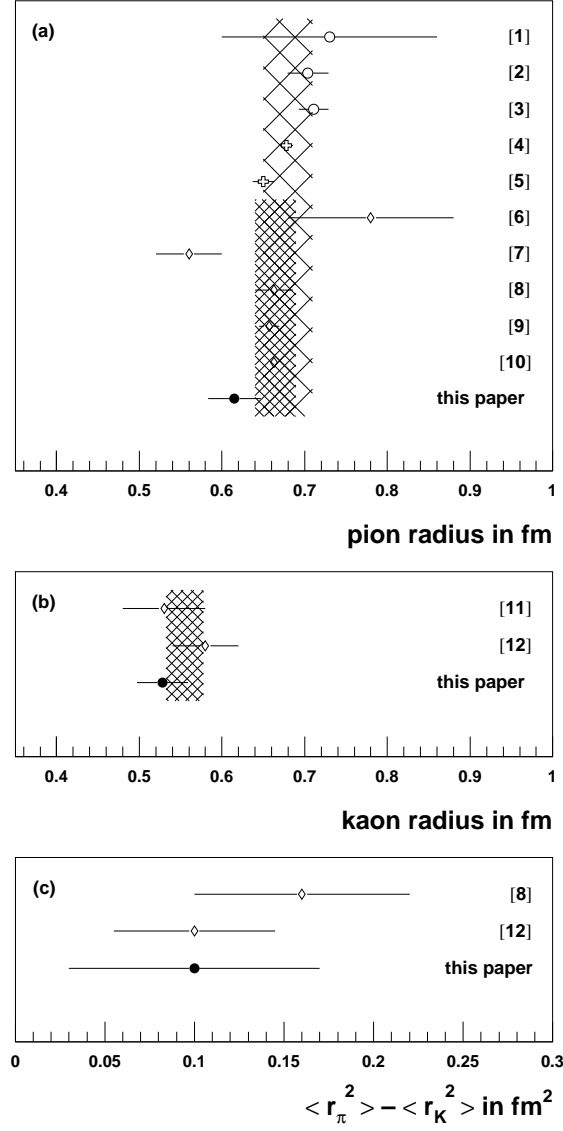


Fig. 4. Experimental values for (a) pion and (b) kaon radius, respectively [1-12]. The average value including the error of all the measurements given in Refs.[1-12] is indicated by the hatched area. The more dense hatch pattern indicates the corresponding average including error obtained from direct $\pi^-e^- \rightarrow \pi^-e^-$ scattering experiments [6-10] and direct $K^-e^- \rightarrow K^-e^-$ scattering experiments [11-12]. (c) Experimental values for the difference of the squared radii of the pion and the kaon [8,12]. The solid points in (a), (b), and (c) are the results obtained in this paper, where the data given in Ref.[18] are used.